Stability: an Abstract Domain for the Trend of Variation of Variables

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Abstract domains for numeric values

precision

Sign

Parity

Constants

```
x = input()

if (x > 10)

  y = 1 / x
```

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  y = 1 / (x - 1) x - 1 = ⊤ ✗
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↓
Intervals
Congruences

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x = input()
⦃x → [−∞, +∞]⦄
if (x > 10)
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```
x = input_array()

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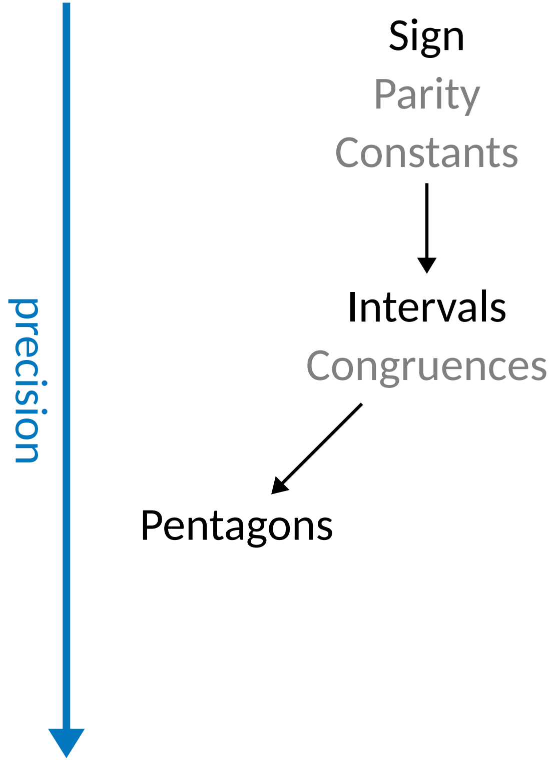


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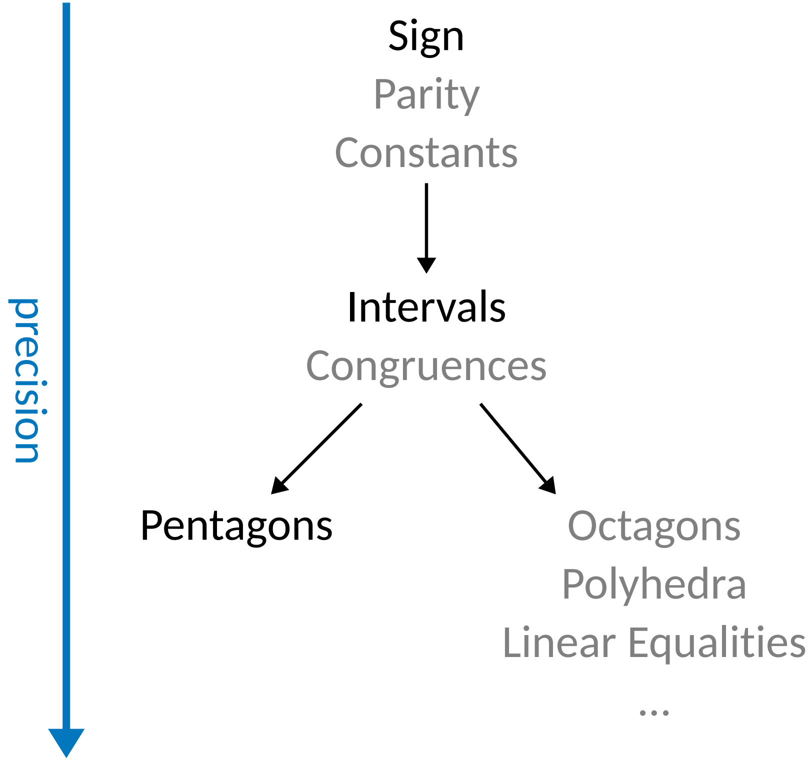


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Where are we aiming?

Non-relational domains

- Rough precision
- Good scalability
- Enough for simple properties

Relational domains

- Higher precision
- Higher computational cost
- Can express more complex invariants

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Stability

- Non-relational (fast)
- Some relational reasoning
- Able to tell something about unknown values

The starting point: a simple Solidity contract

```
contract Coin {
  mapping (address => uint) public balances;
  // [...]
  function send(address dest, uint amount) public {
    require(amount > 0);
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- Problem: `balances[msg.sender]`, `balances[dest]` and `amount` are unknown
- No-go for non-relational analyses: everything is $\top$
- Relational analyses *might* learn something (but not in general)

Our idea: tracking per-variable trends

Keep relations between different values of the same variable:

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▷ No (direct) knowledge on the values of variables needed

▷ Not tailored to specific variable relations

▷ Needs information on some expressions (e.g., sign)

The Stability abstract domain

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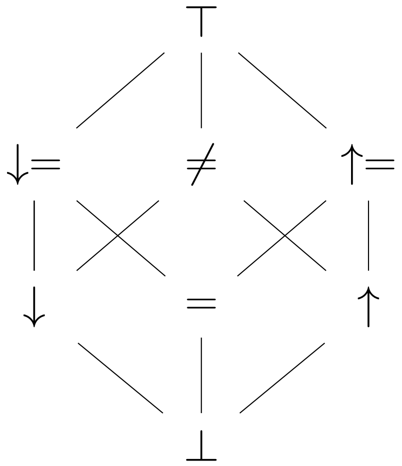
The lattice of per-variable trends:

$\mathsf{Trn} \triangleq$



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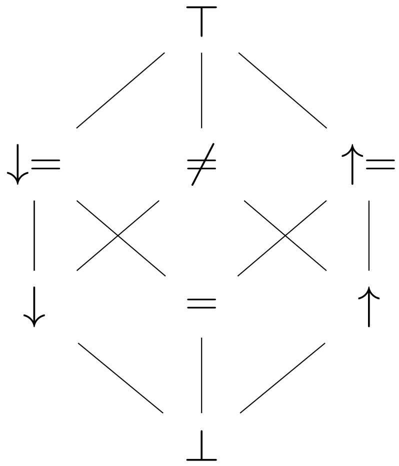

The stability lattice:

$$\mathsf{Stb} \triangleq \langle \mathbb{V}\mathsf{ar} \to \mathsf{Trn}, \dot{\sqsubseteq}_{\mathsf{Trn}}, \dot{\sqcup}_{\mathsf{Trn}}, \dot{\sqcap}_{\mathsf{Trn}}, \bot_{\mathsf{Stb}}, \top_{\mathsf{Stb}} \rangle$$

Stb gives trends to variables w.r.t. **the previous instruction**

The Stability abstract domain (contd.)

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Chaining Stability information

Stb instances can be combined to obtain trends between two arbitrary program points:

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1. Element-wise trend combination:

t_1 \ t_2	$\bot$	$\uparrow=$	$\uparrow$	$=$	$\neq$	$\downarrow$	$\downarrow=$	$\top$
$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$
$\uparrow=$	$\bot$	$\uparrow=$	$\uparrow$	$\uparrow=$	$\top$	$\top$	$\top$	$\top$
$\uparrow$	$\bot$	$\uparrow$	$\uparrow$	$\uparrow$	$\top$	$\top$	$\top$	$\top$
$=$	$\bot$	$\uparrow=$	$\uparrow$	$=$	$\neq$	$\downarrow$	$\downarrow=$	$\top$
$\neq$	$\bot$	$\top$	$\top$	$\neq$	$\top$	$\top$	$\top$	$\top$
$\downarrow$	$\bot$	$\top$	$\top$	$\downarrow$	$\top$	$\downarrow$	$\downarrow$	$\top$
$\downarrow=$	$\bot$	$\top$	$\top$	$\downarrow=$	$\top$	$\downarrow$	$\downarrow=$	$\top$
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$\neq$	$\bot$	$\top$	$\top$	$\neq$	$\top$	$\top$	$\top$	$\top$
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$\downarrow=$	$\bot$	$\top$	$\top$	$\downarrow=$	$\top$	$\downarrow$	$\downarrow=$	$\top$
$\top$	$\bot$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$

2. Combine over different paths with $\dot{\sqcup}_{\text{Trn}}$

Chaining Stability information (contd.)

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The need for additional information

Stability cannot be inferred through stability alone:

- Does `x` increase after `x = x - y`?
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An oracle that knows at least signs of expressions is needed!

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Stability works in **open product** with a numeric domain $\mathcal{A}$:

$$\mathbb{S}^\sharp[\![\ldots]\!] \triangleq \begin{cases} \ldots & \text{if } \mathrm{Q}_{\mathcal{A}}(\mathsf{e}_1) \\ \ldots & \text{if } \mathrm{Q}_{\mathcal{A}}(\mathsf{e}_2) \\ \ldots & \end{cases}$$

The need for additional information

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Stability works in **open product** with a numeric domain $\mathcal{A}$:

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Practical note: no product needed, only a pre-analysis

Examples

...

$\text{Sign} = \{\!| x \to -, y \to 0^{+} |\!\}$ $\text{Stb} = \{\!| x \to =, y \to \downarrow= |\!\}$

```
x = x + 3 * y
```

$\text{Sign} = \{\!| x \to \top, y \to 0^{+} |\!\}$

...

Examples

...

$\text{Sign} = ⟅x \to -, y \to 0^{+}⟆ \quad \text{Stb} = ⟅x \to =, y \to {\downarrow}{=}⟆$

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...

1. Reset all variables to $=$

Examples

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$\text{Sign} = \llparenthesis x \rightarrow -, y \rightarrow 0^{+} \rrparenthesis \quad \text{Stb} = \llparenthesis x \rightarrow =, y \rightarrow \downarrow= \rrparenthesis$

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1. Reset all variables to $=$
2. Query Sign repeatedly

Examples

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Sign = ≀x → −, y → 0⁺)∫   Stb = ≀x → =, y → ↓=∫
x = x + 3 * y
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...
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   2.1. $Q_{\mathcal{A}}(3 * y > 0)$? ✗

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$\text{Sign} = \lbag x \to -, y \to 0^{+} \rbag \quad \text{Stb} = \lbag x \to =, y \to \downarrow= \rbag$

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...

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   2.1. $\mathbb{Q}_{\mathcal{A}}(3 \,*\, y > 0)$? ✗
   2.2. $\mathbb{Q}_{\mathcal{A}}(3 \,*\, y < 0)$? ✗

Examples

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$\text{Sign} = \{x \to -, y \to 0^+\}$ $\text{Stb} = \{x \to =, y \to \downarrow=\}$

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   2.1. $\mathbb{Q}_{\mathcal{A}}(3 * y > 0)$? ✗

   2.2. $\mathbb{Q}_{\mathcal{A}}(3 * y < 0)$? ✗

   2.3. $\mathbb{Q}_{\mathcal{A}}(3 * y == 0)$? ✗

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```
x = x + 3 * y
```

$\text{Sign} = \{\!\!\{ x \to \top, y \to 0^+ \}\!\!\}$ $\text{Stb} = \{\!\!\{ x \to \uparrow=, y \to = \}\!\!\}$

...

1. Reset all variables to $=$
2. Query Sign repeatedly

   2.1. $Q_{\mathcal{A}}(3 * y > 0)$? ✗

   2.2. $Q_{\mathcal{A}}(3 * y < 0)$? ✗

   2.3. $Q_{\mathcal{A}}(3 * y == 0)$? ✗

   2.4. $Q_{\mathcal{A}}(3 * y \geq 0)$? ✓

Examples

```
...
Sign = ⦃x → −, y → 0⁺⦄   Stb = ⦃x → =, y → ↓=⦄
x = x + 3 * y
Sign = ⦃x → ⊤, y → 0⁺⦄   Stb = ⦃x → ↑=, y → =⦄
...
```

1. Reset all variables to $=$

2. Query Sign repeatedly

   2.1. $\mathbb{Q}_{\mathcal{A}}(3 * y > 0)$? ✗

   2.2. $\mathbb{Q}_{\mathcal{A}}(3 * y < 0)$? ✗

   2.3. $\mathbb{Q}_{\mathcal{A}}(3 * y == 0)$? ✗

   2.4. $\mathbb{Q}_{\mathcal{A}}(3 * y \geq 0)$? ✓

   2.5. ...

Examples (contd.)

...

$\text{Sign} = \lbag x \to -, y \to + \rbag \quad \text{Stb} = \lbag x \to =, y \to \downarrow= \rbag$

```
x = x * (y + 2)
```

$\text{Sign} = \lbag x \to -, y \to + \rbag \quad \text{Stb} = \lbag x \to =, y \to = \rbag$

...

1. Reset all variables to $=$
2. Query Sign repeatedly

Examples (contd.)

```
...
Sign = ⦃x → −, y → +⦄  Stb = ⦃x → =, y → ↓=⦄
x = x * (y + 2)
Sign = ⦃x → −, y → +⦄  Stb = ⦃x → =, y → =⦄
...
```

1. Reset all variables to $=$

2. Query Sign repeatedly

   2.1. $Q_{\mathcal{A}}(\mathtt{x} == 0) \vee$
   $Q_{\mathcal{A}}(\mathtt{(y + 2)} == 1)$? ✗

Examples (contd.)

```
...
Sign = ⦇x → −, y → +⦈   Stb = ⦇x → =, y → ↓=⦈
x = x * (y + 2)
Sign = ⦇x → −, y → +⦈   Stb = ⦇x → =, y → =⦈
...
```

1. Reset all variables to $=$
2. Query Sign repeatedly

   2.1. $\mathrm{Q}_{\mathcal{A}}(\mathtt{x == 0}) \vee$
   $\mathrm{Q}_{\mathcal{A}}(\mathtt{(y + 2) == 1})$? ✗

   2.2. $\mathrm{Q}_{\mathcal{A}}(\mathtt{x < 0 \&\& (y + 2) > 1}) \vee$
   $\mathrm{Q}_{\mathcal{A}}(\mathtt{x > 0 \&\& (y + 2) < 1})$? ✗

Examples (contd.)

```
...
Sign = ⦃x → −, y → +⦄   Stb = ⦃x → =, y → ↓=⦄
x = x * (y + 2)
Sign = ⦃x → −, y → +⦄   Stb = ⦃x → =, y → =⦄
...
```

1. Reset all variables to $=$
2. Query Sign repeatedly

   2.1. $Q_{\mathcal{A}}(\texttt{x == 0}) \vee Q_{\mathcal{A}}(\texttt{(y + 2) == 1})$? ✗

   2.2. $Q_{\mathcal{A}}(\texttt{x < 0 \&\& (y + 2) > 1}) \vee Q_{\mathcal{A}}(\texttt{x > 0 \&\& (y + 2) < 1})$? ✗

   2.3. $Q_{\mathcal{A}}(\texttt{x < 0 \&\& (y + 2) < 1}) \vee Q_{\mathcal{A}}(\texttt{x > 0 \&\& (y + 2) > 1})$? ✗

Examples (contd.)

```
...
Sign = ⦇x → −, y → +⦈   Stb = ⦇x → =, y → ↓=⦈
x = x * (y + 2)
Sign = ⦇x → −, y → +⦈   Stb = ⦇x → ⊤, y → =⦈
...
```

1. Reset all variables to $=$
2. Query Sign repeatedly

   2.1. $Q_{\mathcal{A}}(\texttt{x == 0}) \lor Q_{\mathcal{A}}(\texttt{(y + 2) == 1})$? ✗

   2.2. $Q_{\mathcal{A}}(\texttt{x < 0 \&\& (y + 2) > 1}) \lor Q_{\mathcal{A}}(\texttt{x > 0 \&\& (y + 2) < 1})$? ✗

   2.3. $Q_{\mathcal{A}}(\texttt{x < 0 \&\& (y + 2) < 1}) \lor Q_{\mathcal{A}}(\texttt{x > 0 \&\& (y + 2) > 1})$? ✗

   2.4. ...

Examples (contd.)

```
...
Intv = ⦃x → [−12, −3], y → [1, +∞]⦄  Stb = ⦃x → =, y → ↓=⦄
x = x * (y + 2)
Intv = ⦃x → [−∞, −9], y → [1, +∞]⦄  Stb = ⦃x → ↓, y → =⦄
...
```

1. Reset all variables to $=$
2. Query Intv repeatedly

   2.1. $Q_{\mathcal{A}}(\texttt{x == 0}) \vee Q_{\mathcal{A}}(\texttt{(y + 2) == 1})$? ✗

   2.2. $Q_{\mathcal{A}}(\texttt{x < 0 \&\& (y + 2) > 1}) \vee Q_{\mathcal{A}}(\texttt{x > 0 \&\& (y + 2) < 1})$? ✓

   2.3. $Q_{\mathcal{A}}(\texttt{x < 0 \&\& (y + 2) < 1}) \vee Q_{\mathcal{A}}(\texttt{x > 0 \&\& (y + 2) > 1})$? ✗

   2.4. ...

Where can this be applied?

Covariance and Contravariance

```
contract Coin {
  mapping (address => uint) public balances;
  // [...]
  function send(address dest, uint amount) public {
    require(amount > 0);
    require(amount <= balances[msg.sender]);
    balances[msg.sender] -= amount;
    balances[dest] += amount;
    ≀dest → =, amount → =, balances[msg.sender] → ↓, balances[dest] → ↑≀
  }
}
```

▷ Weak relation between pairs/groups of variables

Covariance and Contravariance

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  mapping (address => uint) public balances;
  // [...]
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  }
}
```

▷ Weak relation between pairs/groups of variables

▷ Byproduct of the chained Stability

Covariance and Contravariance

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  }
}
```

- ▷ Weak relation between pairs/groups of variables
- ▷ Byproduct of the chained Stability
- ▷ Can be used to prove functional requirements

Termination (light)

```
int foo(int x, int y) {

  while (x > 0 && y > 0) {

    y = 2 * y;

    x = x - 1;


  }

  return x + y;

}
```

▷ Analyze the program with Stability

Termination (light)

```
int foo(int x, int y) {
  ⦃{x ↦ =, y ↦ =}⦄
  while (x > 0 && y > 0) {

    y = 2 * y;

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  return x + y;

}
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▷ Analyze the program with Stability

Termination (light)

```
int foo(int x, int y) {
  ⟅{x ↦ =, y ↦ =}⟆
  while (x > 0 && y > 0) {
    ⟅{x ↦ =, y ↦ =}⟆
    y = 2 * y;

    x = x - 1;


  }

  return x + y;

}
```

▷ Analyze the program with Stability

Termination (light)

```
int foo(int x, int y) {
  ≀{x ↦ =, y ↦ =}≀
  while (x > 0 && y > 0) {
    ≀{x ↦ =, y ↦ =}≀
    y = 2 * y;
    ≀{x ↦ =, y ↦ ↑}≀
    x = x - 1;


  }

  return x + y;

}
```

▷ Analyze the program with Stability

Termination (light)

```
int foo(int x, int y) {
  ⟅{x ↦ =, y ↦ =}⟆
  while (x > 0 && y > 0) {
    ⟅{x ↦ =, y ↦ =}⟆
    y = 2 * y;
    ⟅{x ↦ =, y ↦ ↑}⟆
    x = x - 1;
    ⟅{x ↦ ↓, y ↦ =}⟆

  }

  return x + y;

}
```

▷ Analyze the program with Stability

Termination (light)

```
int foo(int x, int y) {
  ⦃{x ↦ =, y ↦ =}⦄
  while (x > 0 && y > 0) {
    ⦃{x ↦ ↓=, y ↦ =}⦄
    y = 2 * y;
    ⦃{x ↦ =, y ↦ ↑}⦄
    x = x - 1;
    ⦃{x ↦ ↓, y ↦ =}⦄

  }

  return x + y;

}
```

▷ Analyze the program with Stability

Termination (light)

```
int foo(int x, int y) {
  ≀{x ↦ =, y ↦ =}≀
  while (x > 0 && y > 0) {
    ≀{x ↦ ↓=, y ↦ =}≀
    y = 2 * y;
    ≀{x ↦ =, y ↦ ↑}≀
    x = x - 1;
    ≀{x ↦ ↓, y ↦ =}≀

  }
  ≀{x ↦ =, y ↦ =}≀
  return x + y;

}
```

▷ Analyze the program with Stability

Termination (light)

```
int foo(int x, int y) {
  ⟅{x ↦ =, y ↦ =}⟆
  while (x > 0 && y > 0) {
    ⟅{x ↦ ↓=, y ↦ =}⟆
    y = 2 * y;
    ⟅{x ↦ =, y ↦ ↑}⟆
    x = x - 1;
    ⟅{x ↦ ↓, y ↦ =}⟆

  }
  ⟅{x ↦ =, y ↦ =}⟆
  return x + y;
  ⟅{x ↦ =, y ↦ =}⟆
}
```

▷ Analyze the program with Stability

Termination (light)

```
int foo(int x, int y) {
  ≀{x ↦ =, y ↦ =}∫
  while (x > 0 && y > 0) {
    ≀{x ↦ ↓=, y ↦ =}∫
    y = 2 * y;
    ≀{x ↦ =, y ↦ ↑}∫
    x = x - 1;
    ≀{x ↦ ↓, y ↦ =}∫
    ≀{x ↦ ↓, y ↦ ↑}∫
  }
  ≀{x ↦ =, y ↦ =}∫
  return x + y;
  ≀{x ↦ =, y ↦ =}∫
}
```

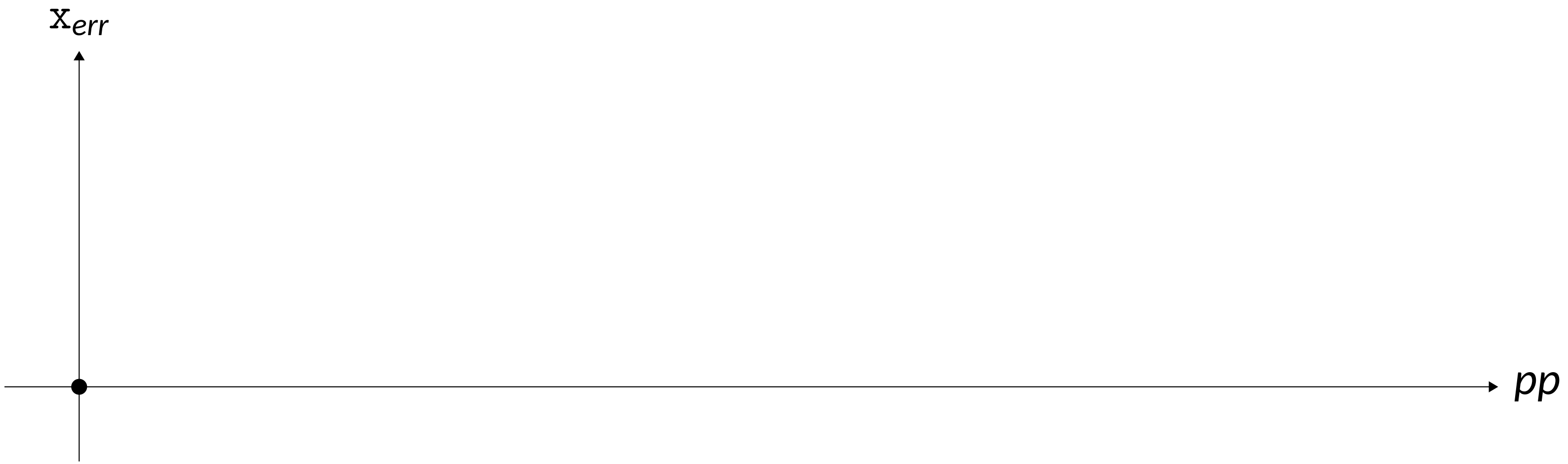
- ▷ Analyze the program with Stability
- ▷ Chain Stability info at end of loops

Termination (light)

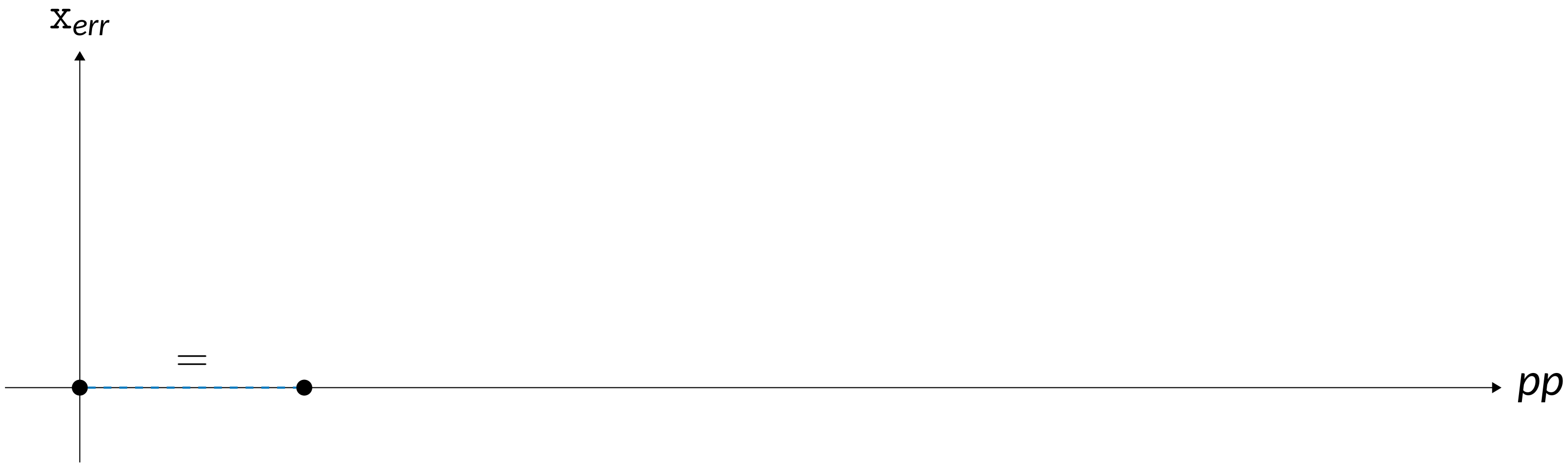
```
int foo(int x, int y) {
  ⦃{x ↦ =, y ↦ =}⦄
  while (x > 0 && y > 0) {
    ⦃{x ↦ ↓=, y ↦ =}⦄
    y = 2 * y;
    ⦃{x ↦ =, y ↦ ↑}⦄
    x = x - 1;
    ⦃{x ↦ ↓, y ↦ =}⦄
    ⦃{x ↦ ↓, y ↦ ↑}⦄
  }
  ⦃{x ↦ =, y ↦ =}⦄
  return x + y;
  ⦃{x ↦ =, y ↦ =}⦄
}
```

- ▷ Analyze the program with Stability
- ▷ Chain Stability info at end of loops
- ▷ Chained information can sometimes prove termination

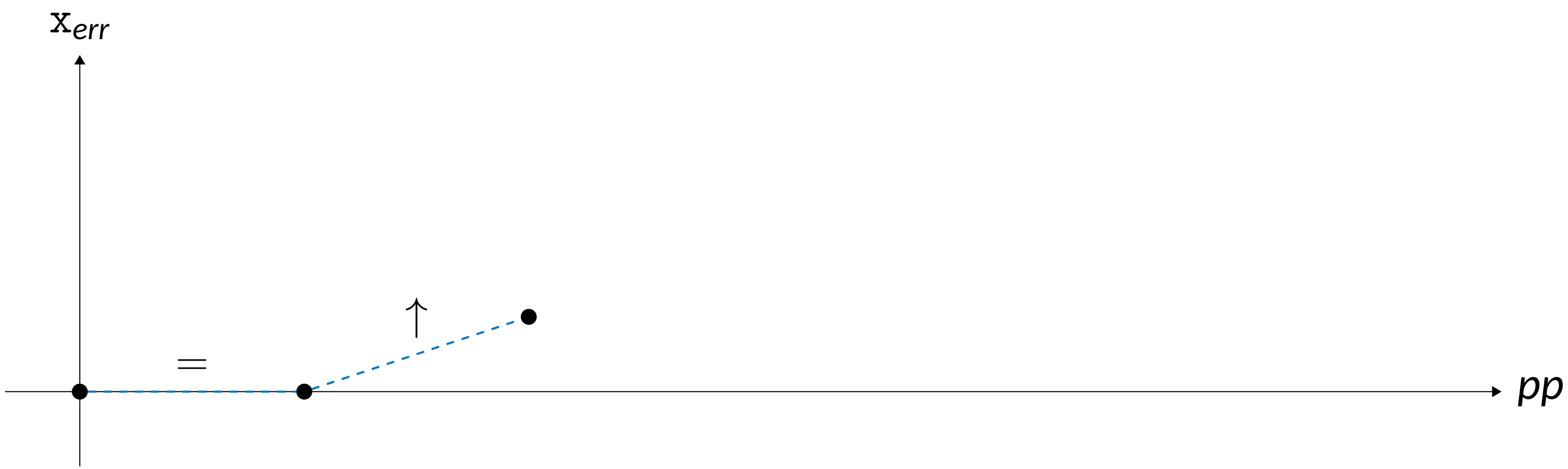
Floating point error



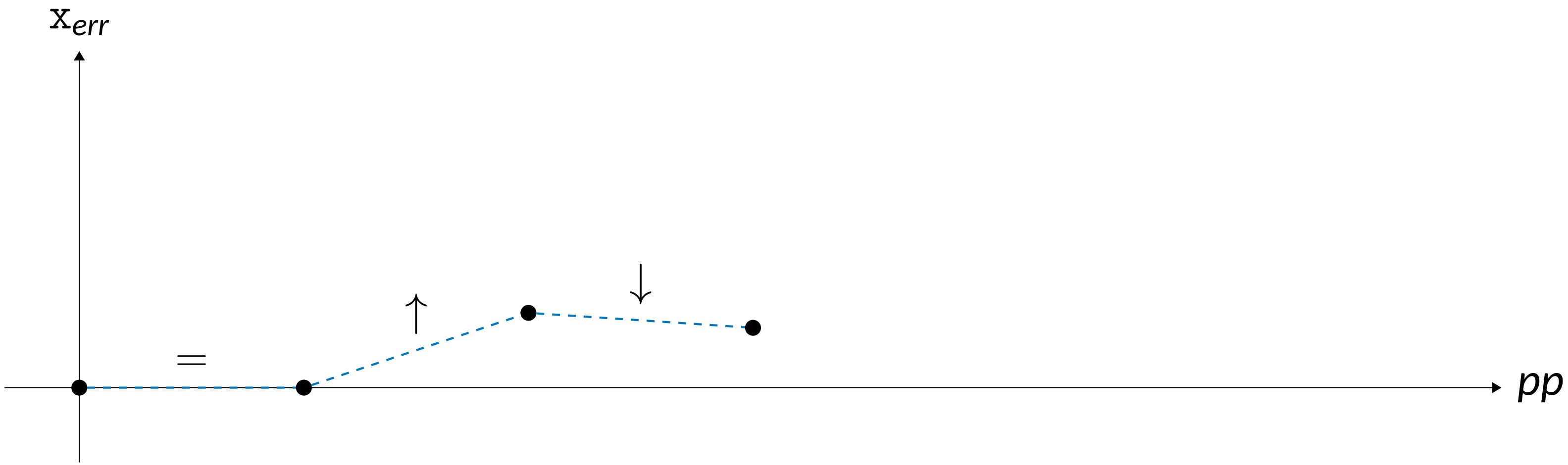
Floating point error



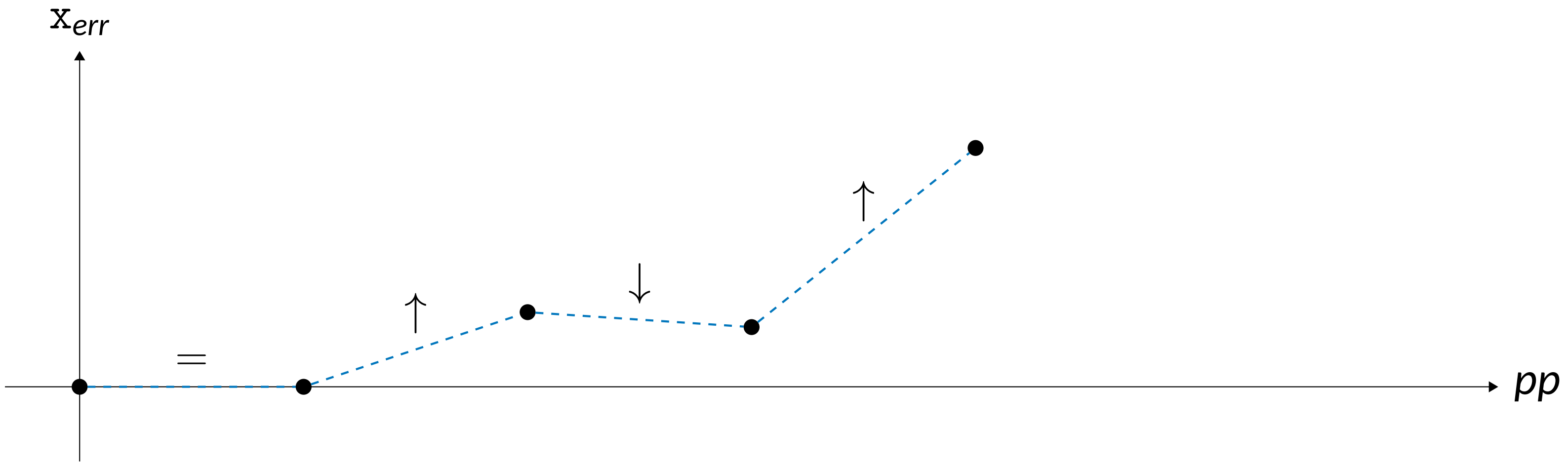
Floating point error



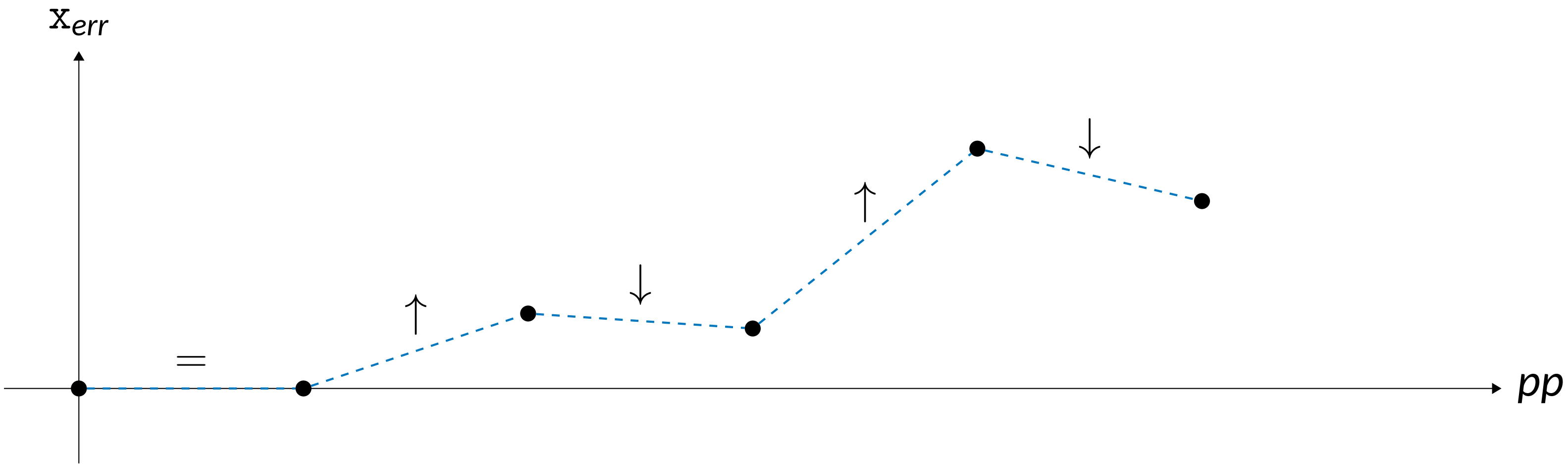
Floating point error



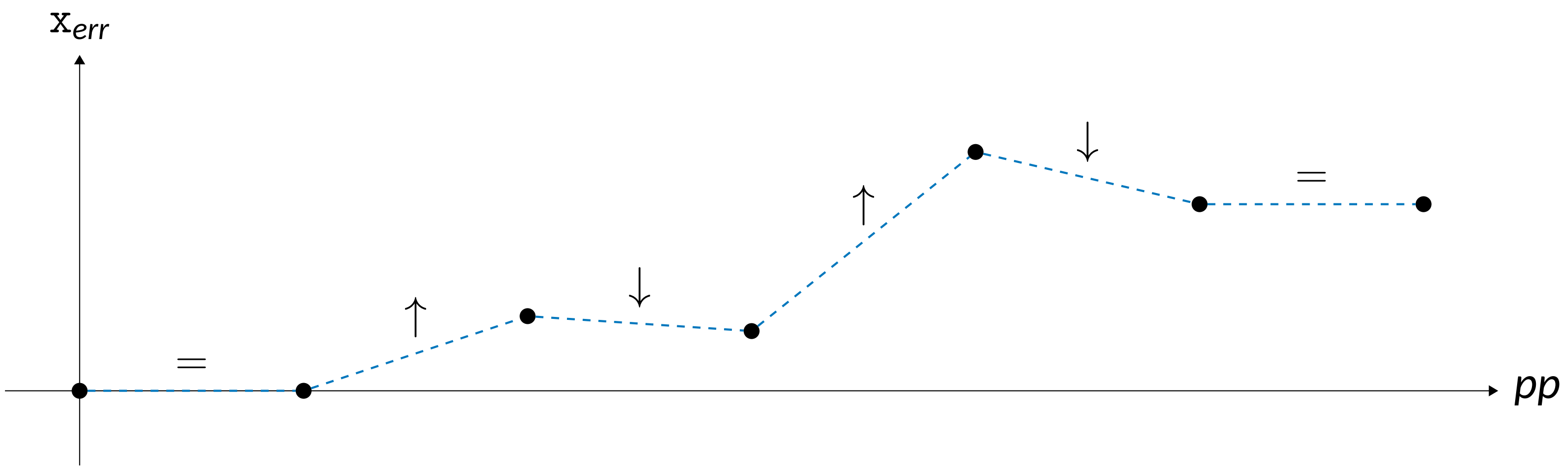
Floating point error



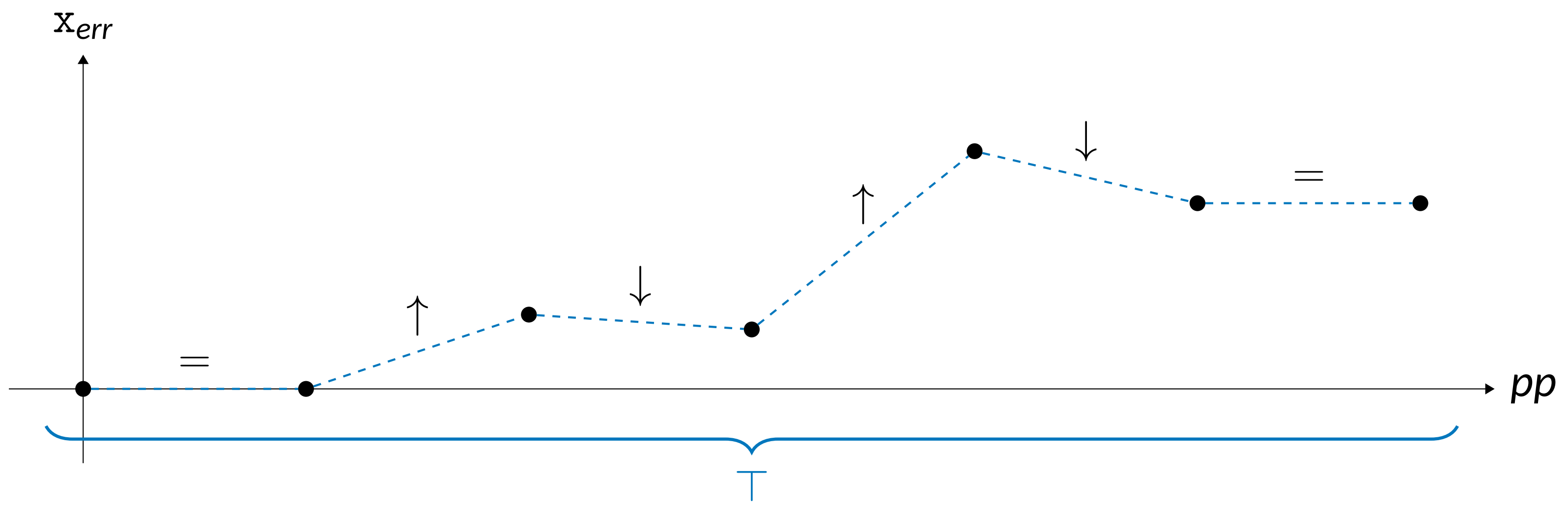
Floating point error



Floating point error



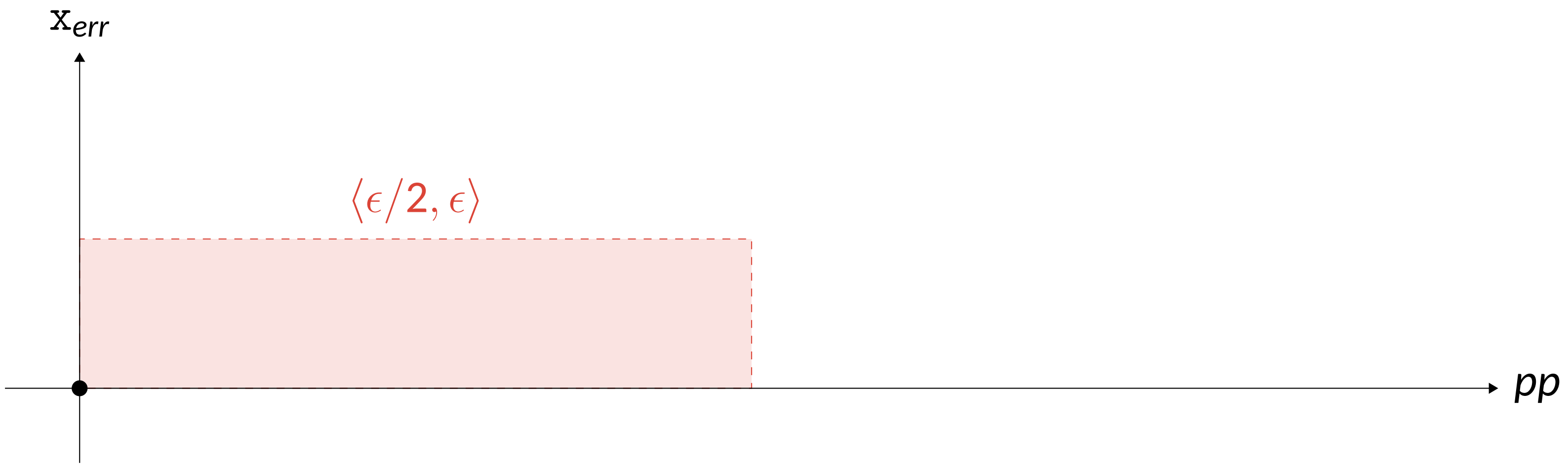
Floating point error



Tiny fluctuations cause the trend to be unstable too quickly

Floating point error (contd.)

Main idea: introduce a sliding tolerance window $\langle \rho, \epsilon \rangle$:



Floating point error (contd.)

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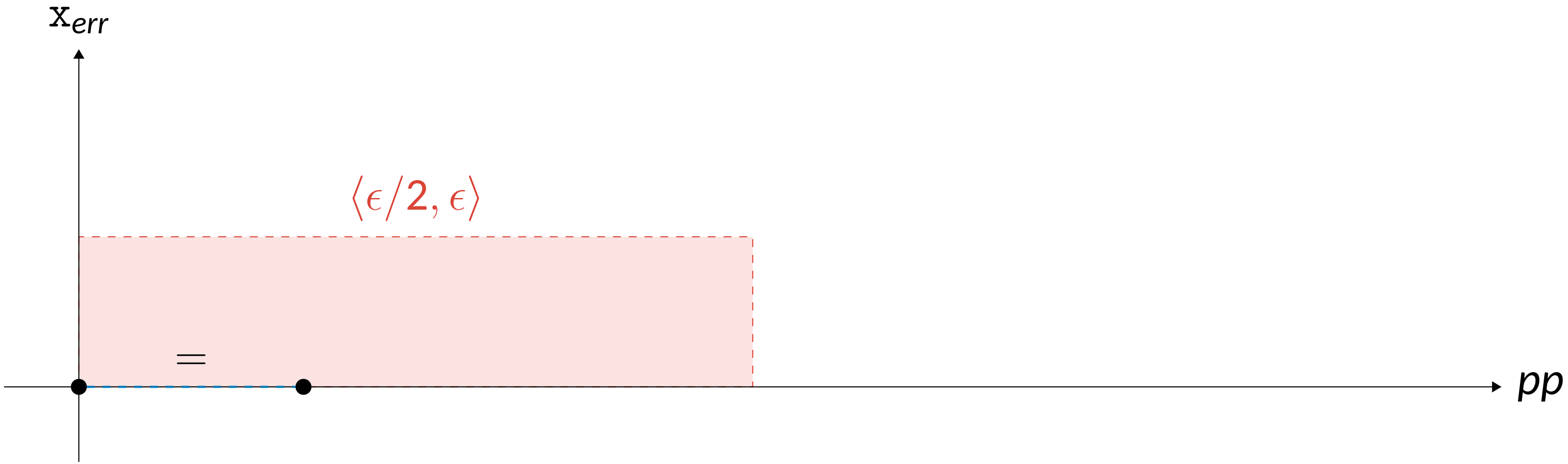

Floating point error (contd.)

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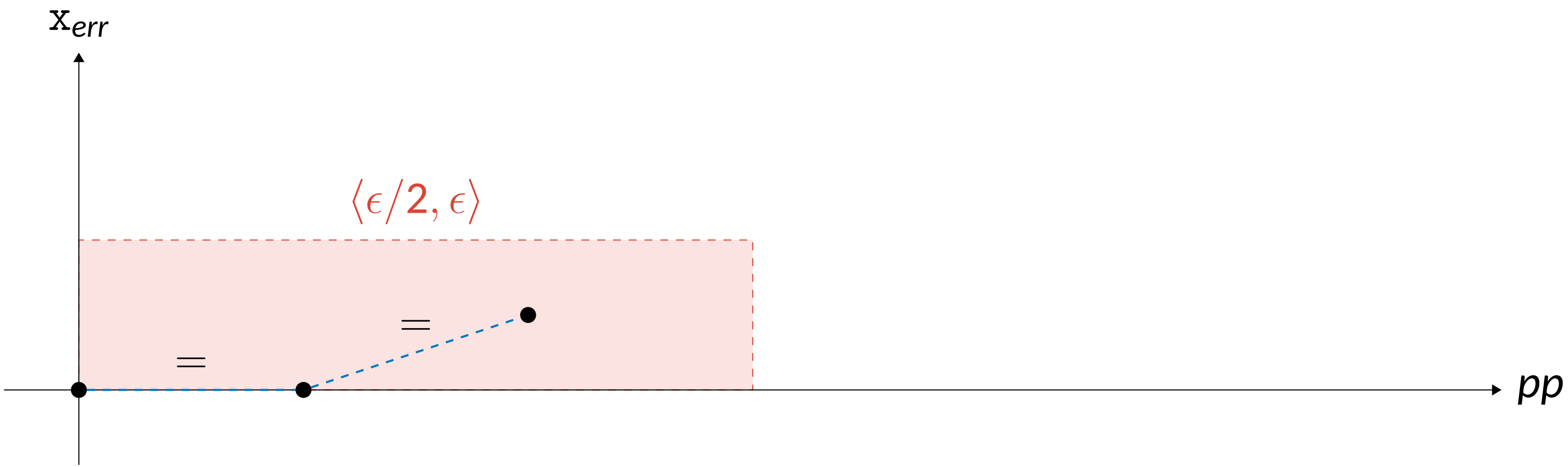

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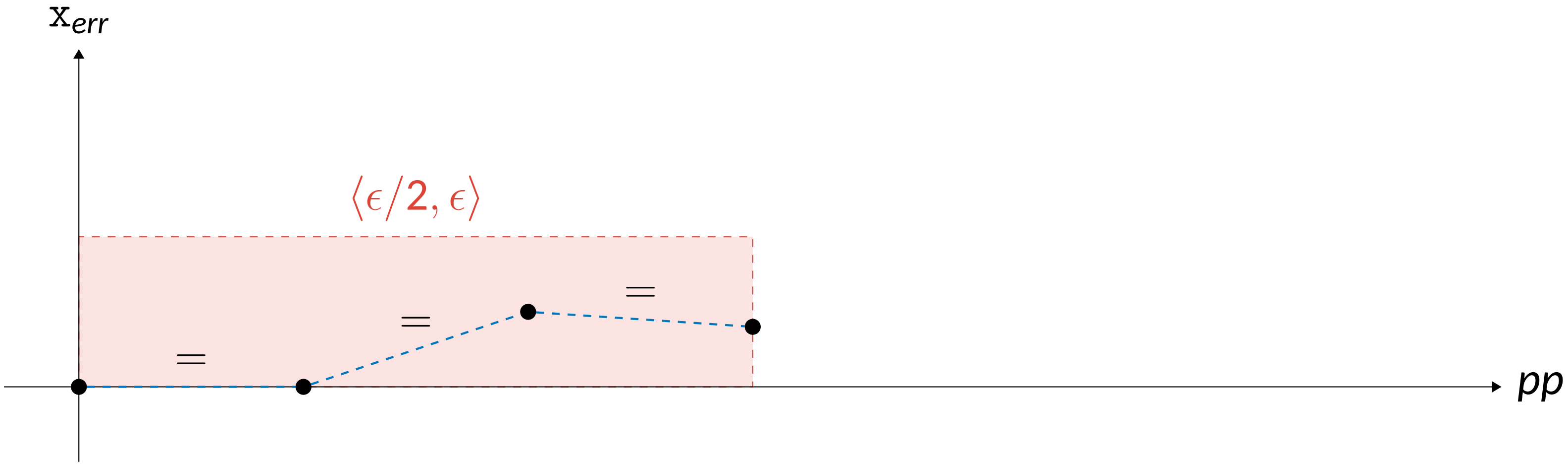

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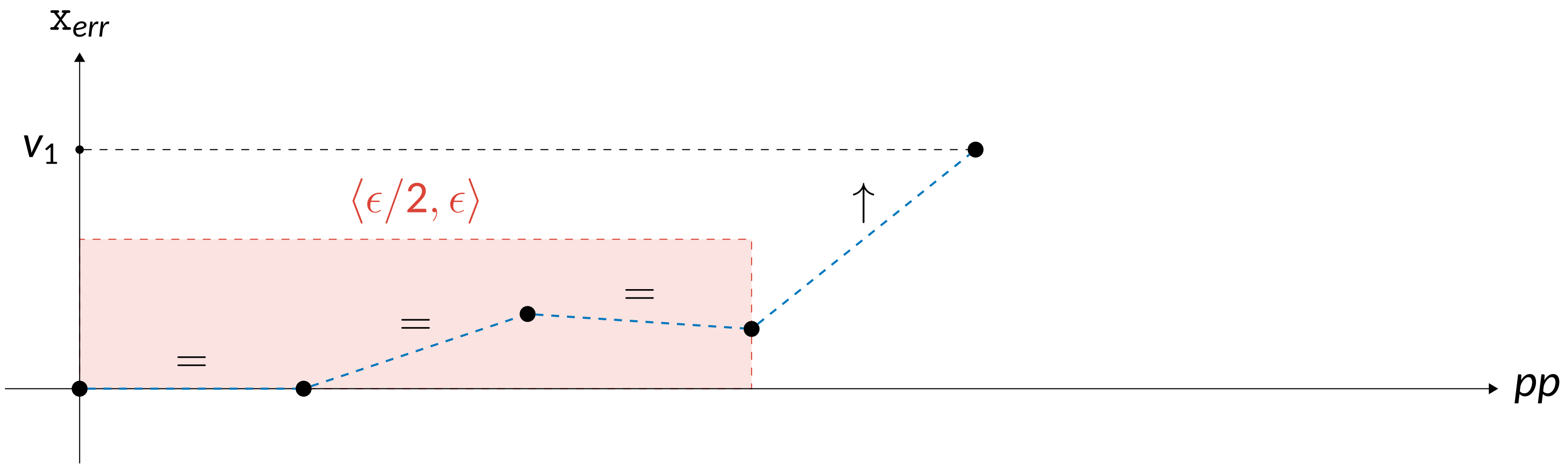

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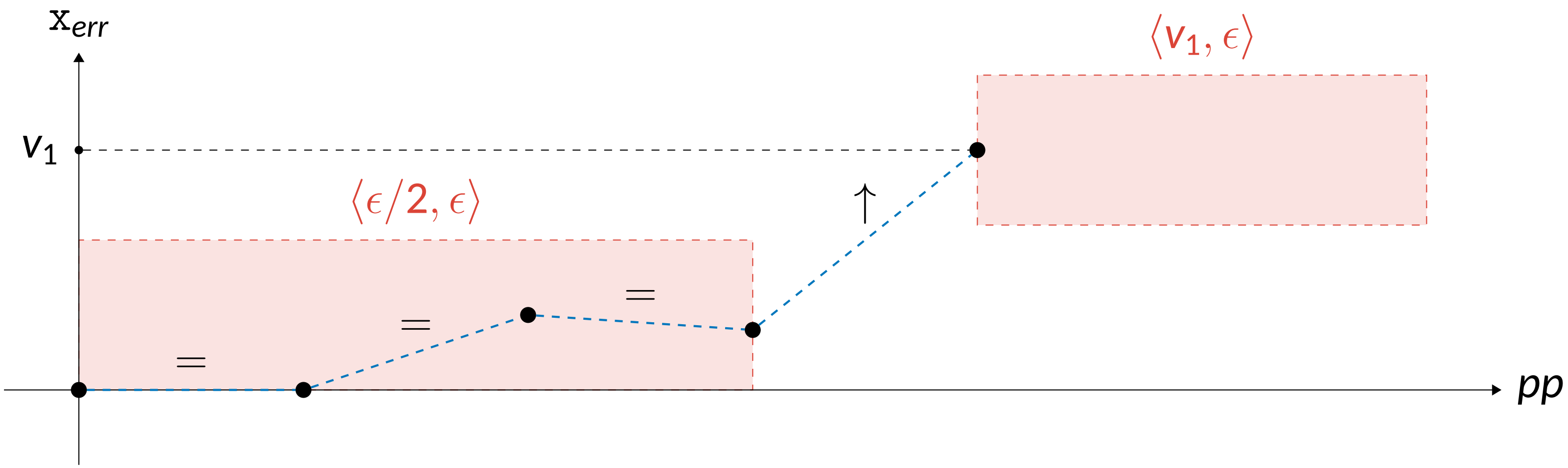

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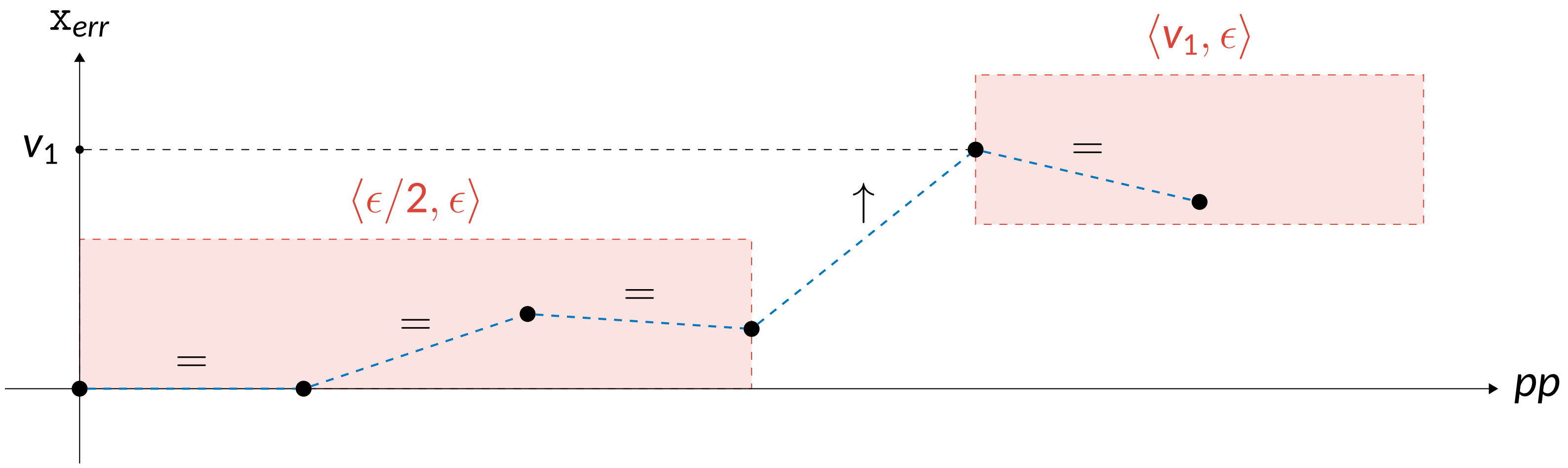

Floating point error (contd.)

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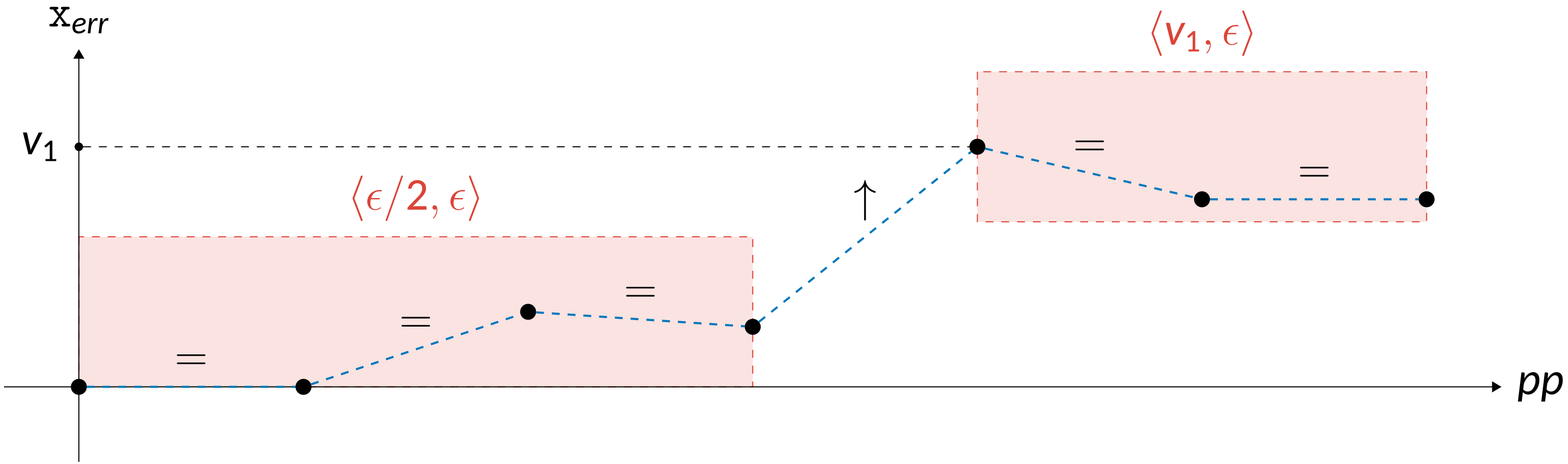

Floating point error (contd.)

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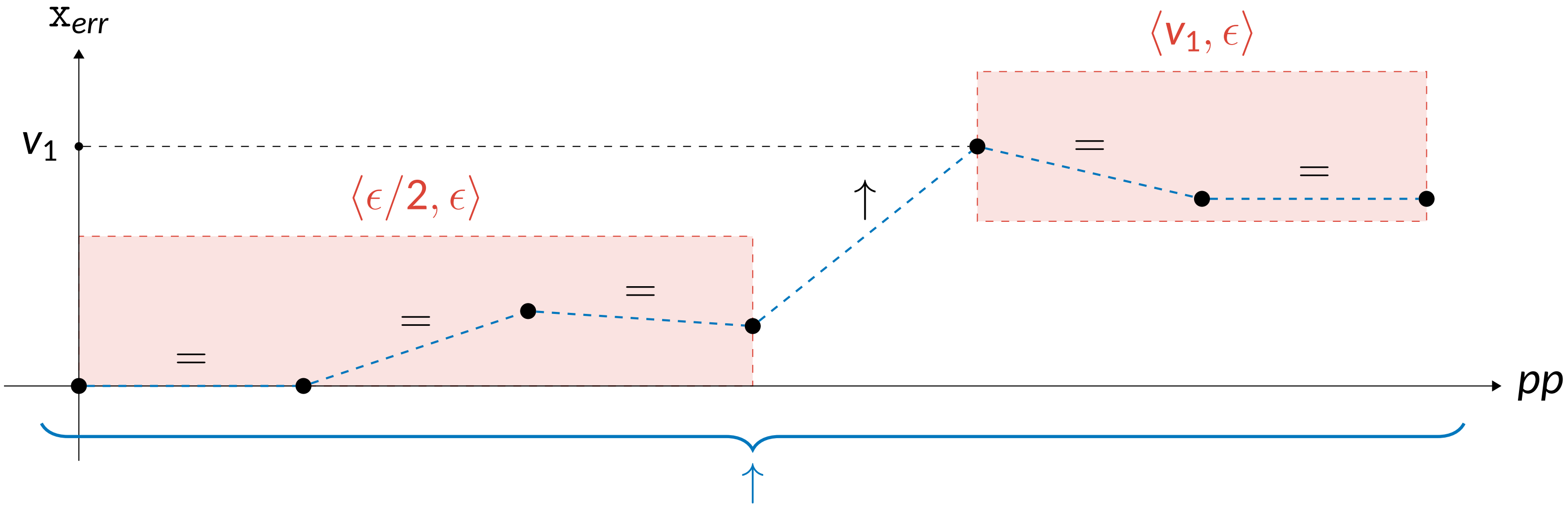

We can show stability up to ϵ!

Summing up

Stability is a new abstract domain for numerical trends

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- Relates values of the same variable at different program points

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Stability is a work in progress

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Stability is a work in progress

- Still searching for a target application

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- Relates values of the same variable at different program points
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- Still searching for a target application
- Benchmarks needed to confirm scalability

Summing up

Stability is a new abstract domain for numerical trends

- Relates values of the same variable at different program points
- Able to track information on $\top$ values for other non-relational domains
- Fast: non-relational and finite

Stability is a work in progress

- Still searching for a target application
- Benchmarks needed to confirm scalability
- Proofs needed once we settle on the semantics

Thanks!

luca.negrini@unive.it